SPACING OPTIMIZATION OF ARRAYS ABOVE AN IMPERFECTLY CONDUCTING GROUND AS AN INVERSE PROBLEM

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ABSTRACT

The paper presents an analytical approach to an optimization of spacing and excitation of arrays generating a given directional pattern above an imperfectly conducting ground plane. The proposed method consists in two step consecutive approximation procedure in which Newton-Raphson and or Penrose-Moore or Tikhonov regularization techniques are used.

It has been known for a long time [6] that the coordinates of radiating elements can be used as an additional set of parameters for advanced array design. However, due to the nonlinear character of the dependence of the directional function on the location of the elements, obvious difficulties are met in treating the problem analytically. Effectively applied numerical methods make use of dynamic programming or other systematic search procedures [5],[4] and [7]. As of this time the solutions to the problem are still far from being satisfactory [1], [3]. Much more complicated and less elucidated in the literature is the problem of spacing optimization applied to arrays generating a given directional pattern above an imperfectly conducting ground.

We present here an attempt to approach this problem analytically considering it as unit inverse problem. It results in the two step consecutive approximation procedure in which Newton-Raphson and or Penrose-Moore, or Tikhonov regularization techniques are used.

We consider line currents I_{nx} located on the hights h_n along the OZ axis above an imperfectly conducting plane S, fig.1.

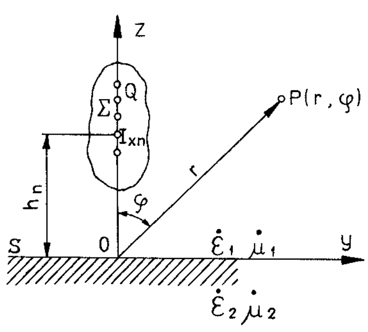

FIGURE 1: Line sources above the ground.

We assume that the currents do not vary along the X coordinate, and that the approximate boundary conditions of Grinberg and Fok [2] apply on the plane S. Then, according to [8], the matrix relation

$$(1)\qquad (a_m) = (k_{mn}) \cdot (c_n) \; ; \; \begin{matrix}(m=1,3,5,\ldots \infty)\\(n=1,2,3,\ldots N)\end{matrix}$$

relates the complex amplitudes c_n of the currents with their complex directional function given by

$$(2)\qquad F(\varphi) = \sum_{m=1,3,\ldots}^{\infty} a_m \cos(m\varphi)$$

where in general a_m assume complex values and

$$(3)\qquad k_{mn} = g_{mn} - f_{mn}$$

describe the properties of the sources themselves. The matrix G depends on the location of currents and represents their contribution to the far field when $E_{x|S} = 0$, what corresponds to the perfect conductivity of the ground. The matrix F depends also on the ground parameters and represents the correction due to the actual boundary value distribution $E_{x|S}$. We note that according to [8]

$$(4)\qquad g_{mn} = (-1)^{\frac{m-1}{2}} J_m(H_n)$$

where J_m is the Bessel function and $H_n = kh_n$ is the coordinate of the n-th current over the ground, while

$$(5)\qquad f_{mn} = -i \int_{-1}^{1} \frac{e^{iH_nW}}{W+N} \cos(m \arcsin(u))du$$

where, $W = \sqrt{1-u^2}$, $N = \sqrt{\dot{\varepsilon}_2' \dot{\mu}_1' / \dot{\varepsilon}_1' \dot{\mu}_2'}$, $\dot{\varepsilon}'$ and $\dot{\mu}'$ denote the complex relative electric and magnetic permeability of the media correspondingly.

We assume that the initial set of H_n, which was chosen according to the secondary requirements such as feeding simplicity or advantages in design, is perturbed so that each H_n gets an increment of x_n. Then we rewrite (1) expanding the matrix K with respect to the powers of $\mathcal{X} = (x_n)$.

Retaining only the linear term we have

$$(6)\qquad A = K_{\mathcal{X}=0} \cdot C + J[K]_{\mathcal{X}=0} \cdot D(c_n) \cdot \mathcal{X}$$

CH1355-7/78/0000-0204$00.75

where $J[K]$ is the Jacobian matrix and D denotes the diagonal matrix. We note that the derivatives of the Bessel functions are expressible also by Bessel functions. For the linear independence of them $J[K]$, as well as K, is the maximal rank matrix and their rank $r(J[K]) = N$, where N is the number of array elements.

In the case which is of special interest to us the directional function $F(\varphi)$ is considered as known and represented by the a priori given matrix A while the optimal excitation matrix C and the optimal position increment matrix X are to be determined.

It is known [8] that for the initial set of H_n, (1) should be considered as an overdetermined system of linear equations with respect to C. It always has the generalized solution (Penrose-Moore), as well as the regular solution according to Tikhonov. Now, for the fixed C relation (6) in turn can be considered as an overdetermined system of equations for X which by virtue of the maximal rank of $J[K]$ has the generalized and Tikhonov solutions too. This gives the possibility of proposing the two step consecutive approximation procedure for finding the optimal excitation and spacing for an array generating a given directional function above an imperfectly conducting ground.

In the first step of the consecutive $\nu+1$ approximation, with fixed $C=C^{\nu}$ we optimize the array spacing by calculating

$$H_n^{\nu+1} = H_n^{\nu} + x_n^{\nu+1} \tag{7}$$

where $x_n^{\nu+1}$ are found by applying jointly the Newton-Raphson technique and, or generalized solution or Tikhonov regularization method to (6).

$$X^{\nu+1} = \left\{ \left[J[K^{\nu}]\cdot D(c_n^{\nu})\right]^* P^{\nu}\left[J[K^{\nu}]\cdot D(c_n^{\nu})\right] + \beta^{\nu+1} I\right\}^{-1} \cdot \left[J[K^{\nu}]\cdot D(c_n^{\nu})\right]^* P^{\nu}\left[A - K^{\nu}C^{\nu}\right] \tag{8}$$

Thus, $X^{\nu+1}$ having been found minimizes with the acuracy to the second order terms the functional

$$\triangle_{\beta}^{\nu+1} = \left\| A - K^{\nu+1}C^{\nu} \right\| + \beta^{\nu+1}\left\|X^{\nu+1}\right\| \tag{9}$$

In the second step, to prevent the accumulation of the errors in the excitation matrix C, resulting from the perturbation of the location of the elements in the first step, we calculate with fixed $X = X^{\nu+1}$ the corrected excitation matrix

$$C^{\nu+1} = \left[(K^{\nu+1})^* P^{\nu+1} K^{\nu+1} + \alpha^{\nu+1} I\right]^{-1} \cdot (K^{\nu+1})^* P^{\nu+1} A \tag{10}$$

which exactly minimizes the functional

$$\triangle_{\alpha}^{\nu+1} = \left\| A - K^{\nu+1}C^{\nu+1} \right\| + \alpha^{\nu+1}\left\|C^{\nu+1}\right\| \tag{11}$$

where P^{ν}, α^{ν} and β^{ν} denote the diagonal matrix of weights and regularization parameters used in the ν-th approximation. When regularization parameters tend to zero the Tikhonov's solutions (8) and (10) go over to generalized solutions.

The convergence of the method depends on the sensitivity of the system to the deviation of the excitation matrix from it's exact value, caused by the perturbation of the elements position in the first step of each consecutive approximation. However, it is expected that properly choosing the regularization parameters $_{\nu}\beta^{\nu}$ one can hold the position increments X^{ν} within the range of convergence of the method. The corresponding numerical example is under preparation.

The proposed method results in a far going optimization of arrays. As a result a substantial decrease of the number of elements and dimensions of arrays is expected.

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